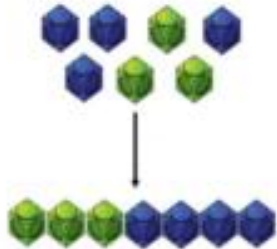
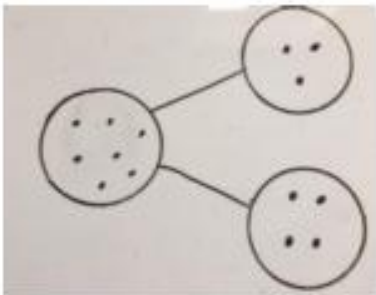
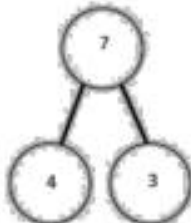
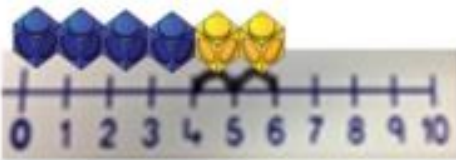
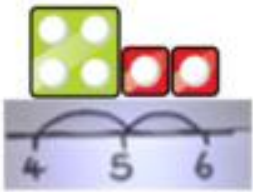
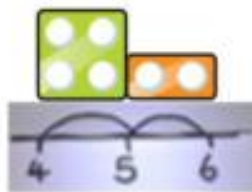
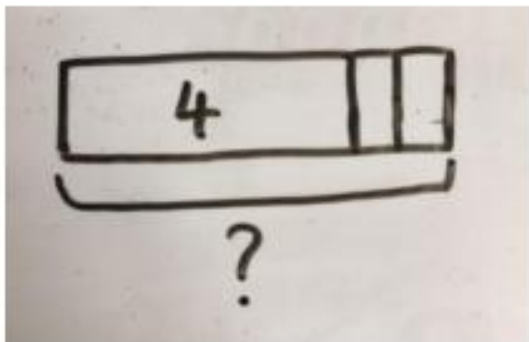

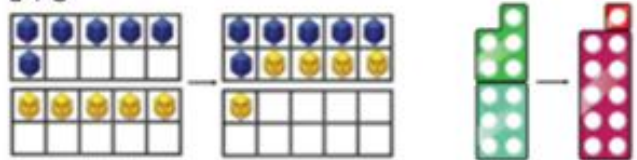
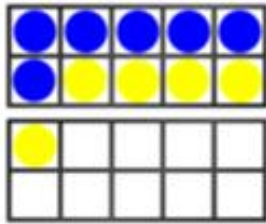
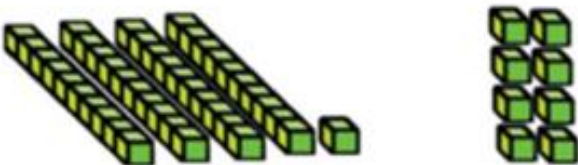
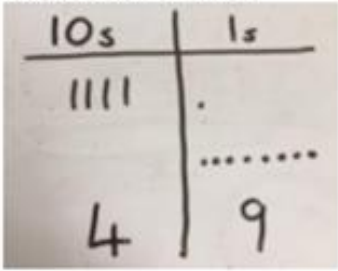
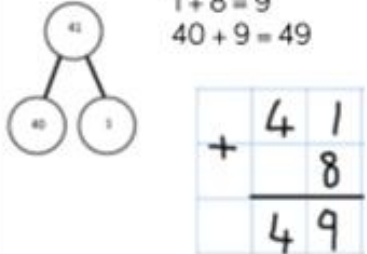
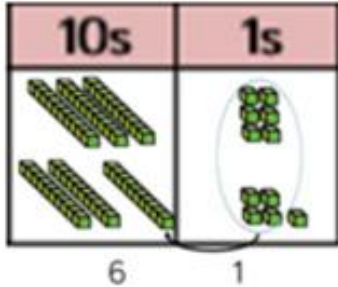
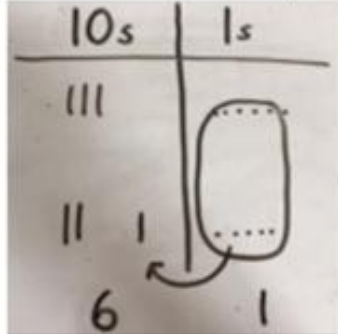


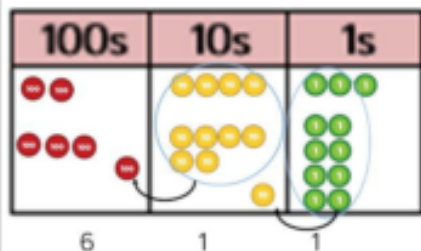
Addition

Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 
<p>Counting on using number lines using cubes or Numicon.</p>   	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 

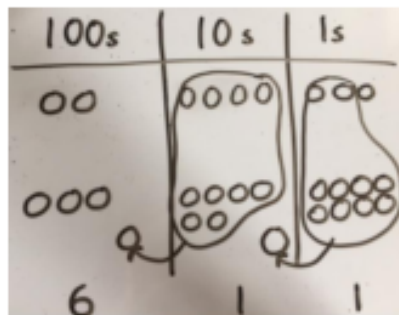


<p>Regrouping to make 10; using ten frames and counters/cubes or using Numicon.</p> <p>6 + 5</p> 	<p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to develop an understanding of equality e.g.</p> $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
<p>TO + O using base 10. Continue to develop understanding of partitioning and place value.</p> <p>41 + 8</p> 	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	<p>41 + 8</p> <p>1 + 8 = 9 40 + 9 = 49</p> 
<p>TO + TO using base 10. Continue to develop understanding of partitioning and place value.</p> <p>36 + 25</p> 	<p>Children to represent the base 10 in a place value chart.</p> 	<p>Looking for ways to make 10.</p> <p>36 + 25 =</p> <p>30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61</p> <p>1 5</p> <p>Formal method:</p> $\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline \end{array}$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



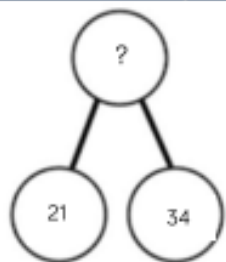
Children to represent the counters in a place value chart, circling when they make an exchange.



243

$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 11 \end{array}$$

Conceptual variation; different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

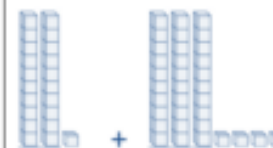
$$21 + 34 = 55. \text{ Prove it}$$

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\boxed{} = 21 + 34$$

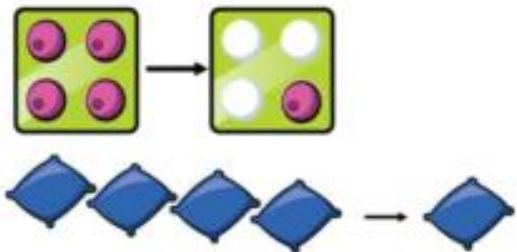
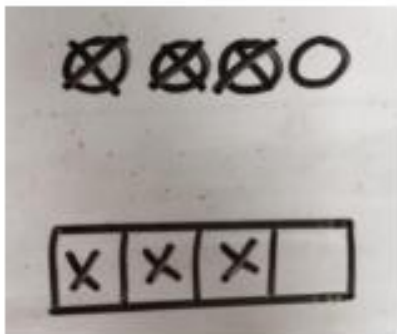
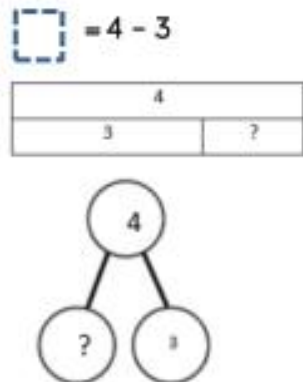
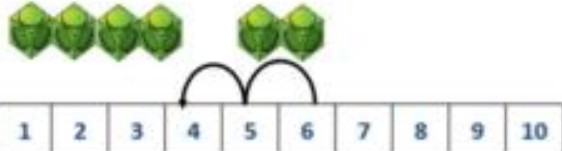
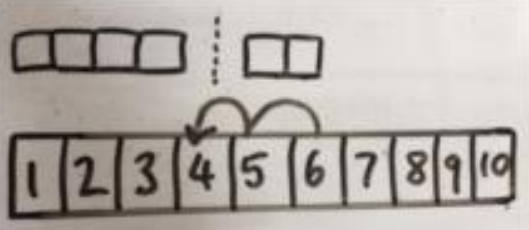
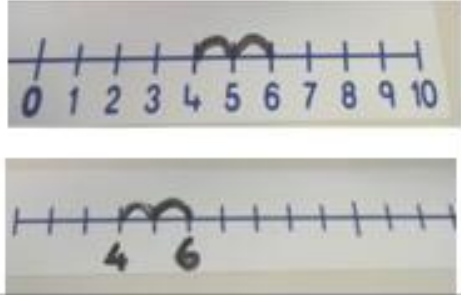
Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

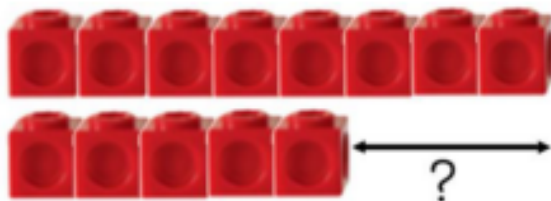
10s	1s
2	1
3	?
?	5

Subtraction

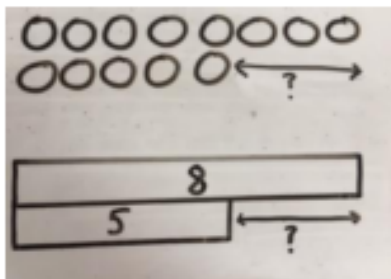
Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p></p>
<p>Counting back (using number lines or number tracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



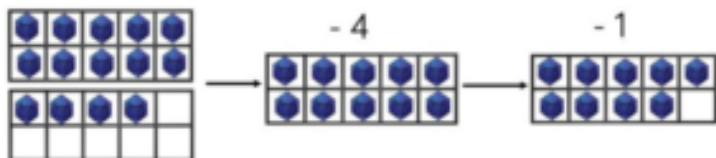
Find the difference between 8 and 5.

8 - 5, the difference is

Children to explore why
 $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

14 - 5



Children to present the ten frame pictorially and discuss what they did to make 10.



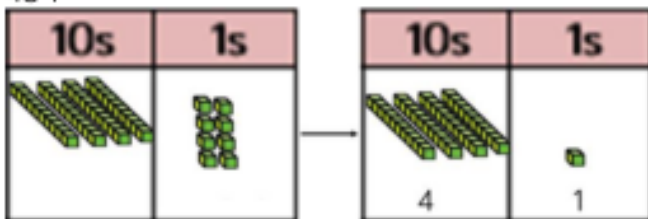
Children to show how they can make 10 by partitioning the subtrahend.

$$\begin{array}{r} 14 - 5 = 9 \\ \swarrow \quad \searrow \\ 4 \quad \quad 1 \end{array}$$

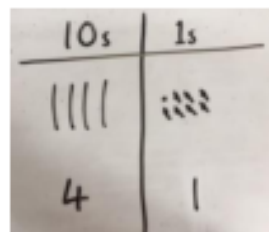
$$\begin{array}{l} 14 - 4 = 10 \\ 10 - 1 = 9 \end{array}$$

Column method using base 10.

48-7

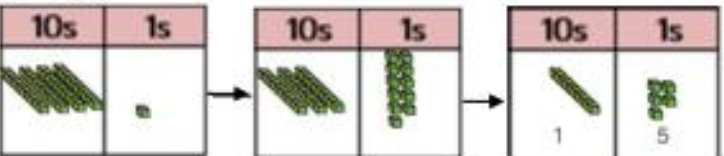


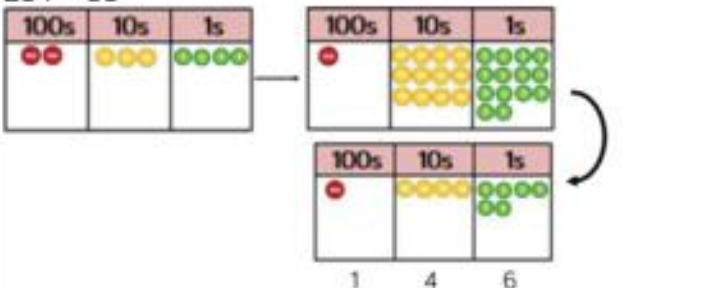
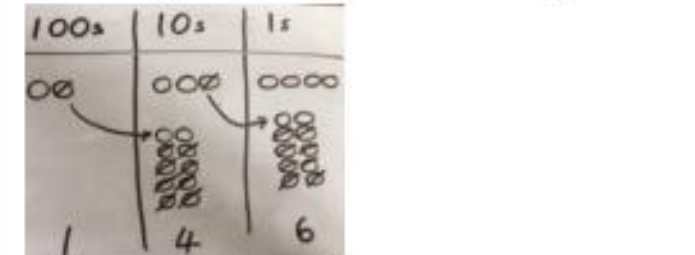
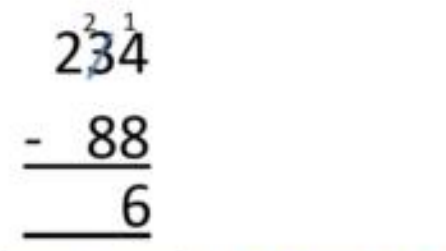

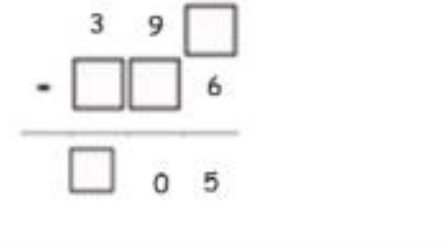


Children to represent the base 10 pictorially.

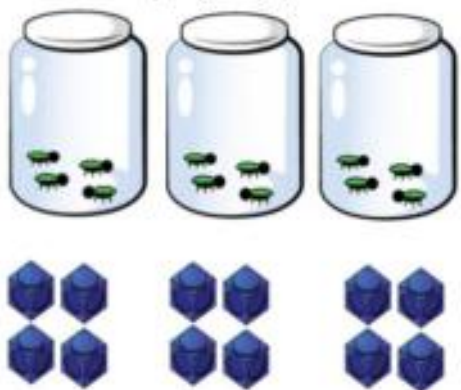
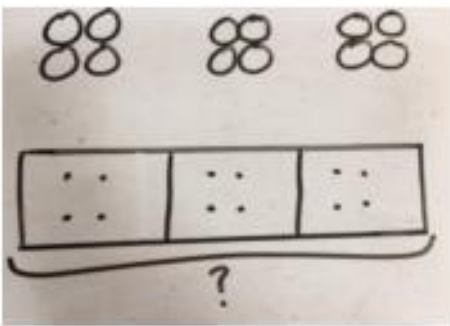
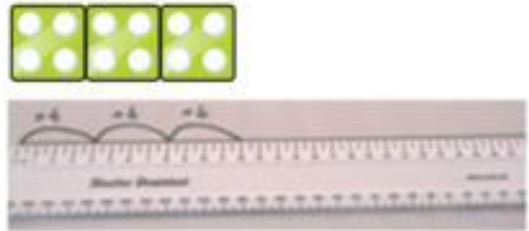
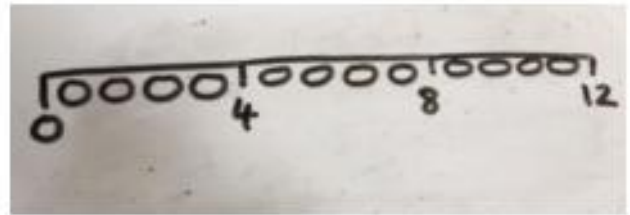
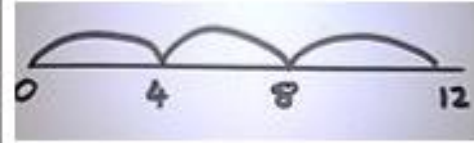


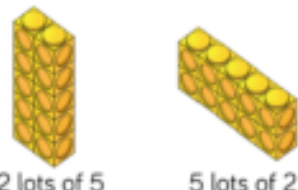
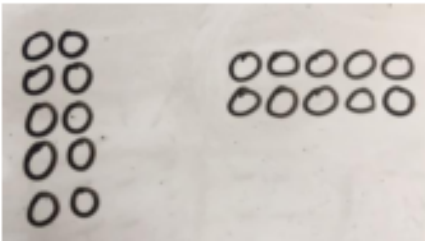
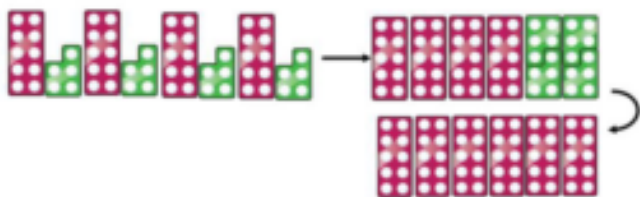
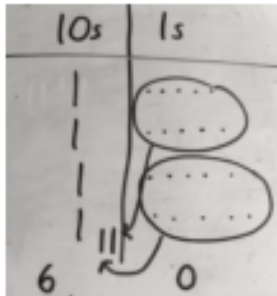
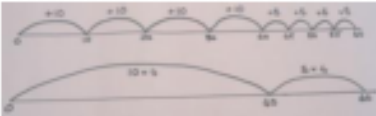




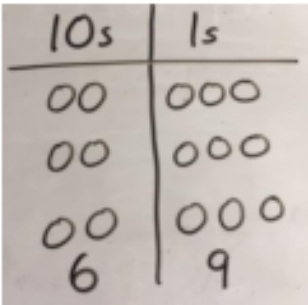


Column method or children could count back 7.

	4	8
-		7
	4	1

<p>Column method using base 10 and having to exchange. 41 - 26</p> 	<p>Represent the base 10 pictorially, remembering to show the exchange.</p> 	<p>Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.</p> 							
<p>Column method using place value counters. 234 - 88</p> 	<p>Represent the place value counters pictorially; remembering to show what has been exchanged.</p> 	<p>Formal column method. Children must understand what has happened when they have crossed out digits.</p> 							
<p>Conceptual variation; different ways to ask children to solve 391 - 186</p>									
 <table border="1" data-bbox="143 1264 647 1364"> <tr> <td></td> <td>391</td> <td></td> </tr> <tr> <td>186</td> <td></td> <td>?</td> </tr> </table>		391		186		?	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>Calculate the difference between 391 and 186.</p>	<p>$\square = 391 - 186$</p> <p>391 -186 —</p> <p>What is 186 less than 391?</p>	<p>Missing digit calculations</p> 
	391								
186		?							

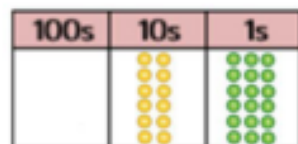
Multiplication

Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Number lines to show repeated groups- 3×4</p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p> 

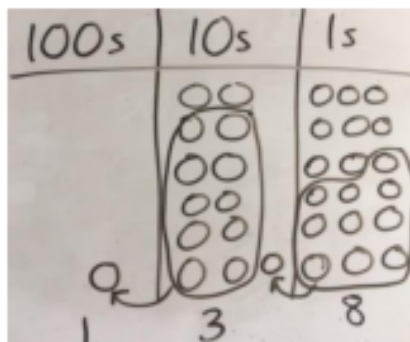
<p>Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$</p>  <p>2 lots of 5 5 lots of 2</p>	<p>Children to represent the arrays pictorially.</p> 	<p>Children to be able to use an array to write a range of calculations e.g.</p> <p>$10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$</p>				
<p>Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15</p> 	<p>Children to represent the concrete manipulatives pictorially.</p> 	<p>Children to be encouraged to show the steps they have taken.</p> <p>4×15 $\swarrow \searrow$ 10 5</p> <p>$10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$</p> <p>A number line can also be used</p> 				
<p>Formal column method with place value counters (base 10 can also be used.) 3×23</p> <table border="1" data-bbox="150 1099 452 1327"><thead><tr><th>10s</th><th>1s</th></tr></thead><tbody><tr><td></td><td></td></tr></tbody></table> <p>6 9</p>	10s	1s			<p>Children to represent the counters pictorially.</p> 	<p>Children to record what it is they are doing to show understanding.</p> <p>3×23 $3 \times 20 = 60$ $\swarrow \searrow$ $3 \times 3 = 9$ 20 3 $60 + 9 = 69$</p> <p>$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$</p>
10s	1s					
						

Formal column method with place value counters.

$$6 \times 23$$



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

$$6 \times 23 =$$

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

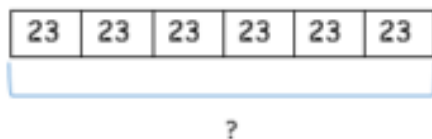
To get 744 children have solved 6×124 .

To get 2480 they have solved 20×124 .

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

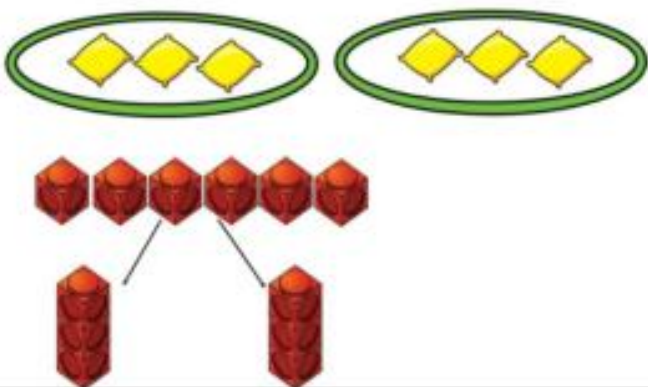
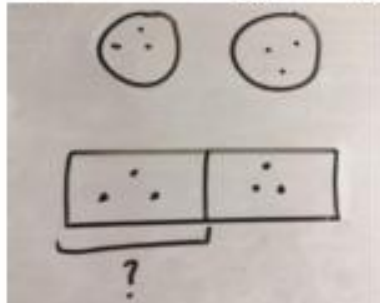
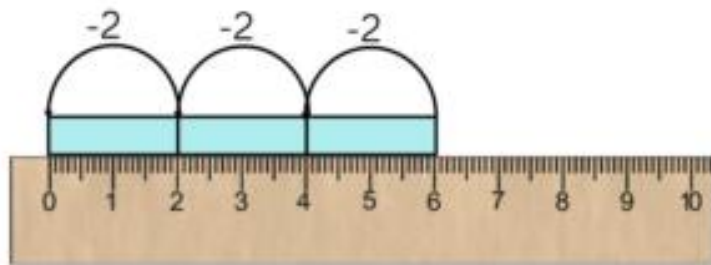
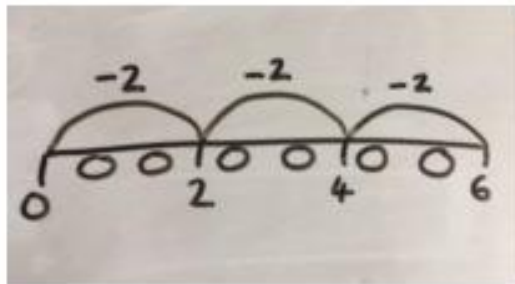
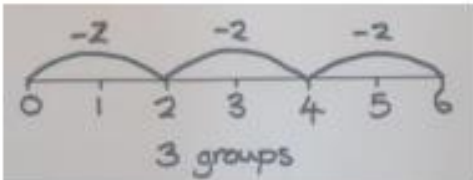
$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times 23 \quad \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?



Division

Concrete	Pictorial	Abstract		
<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p> 	<p>$6 \div 2 = 3$</p> <table border="1"><tr><td>3</td><td>3</td></tr></table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			
<p>Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$</p>  <p>3 groups of 2</p>	<p>Children to represent repeated subtraction pictorially.</p> 	<p>Abstract number line to represent the equal groups that have been subtracted.</p>  <p>3 groups</p>		

2d + 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.

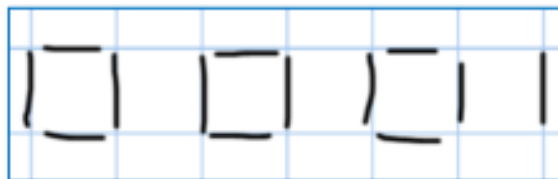
$$13 \div 4$$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

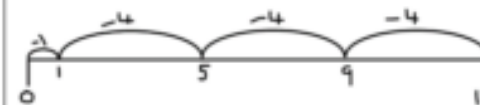


There are 3 whole squares, with 1 left over.

$$13 \div 4 = 3 \text{ remainder } 1$$

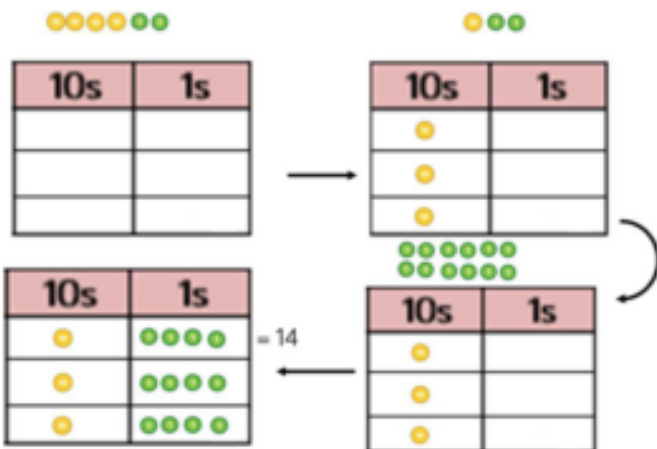
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

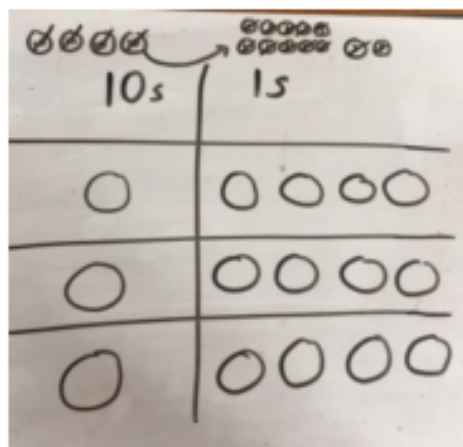


Sharing using place value counters.

$$42 \div 3 = 14$$



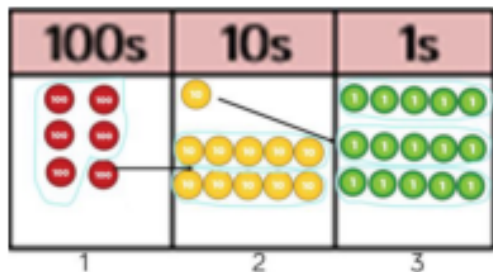
Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

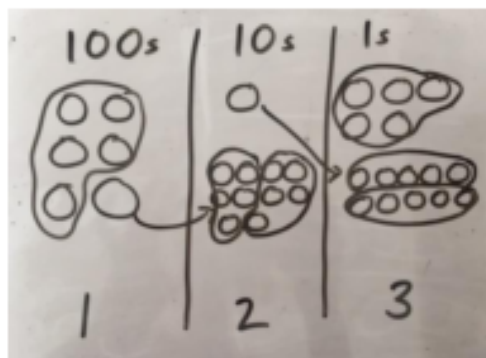
$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

Short division using place value counters to group.
 $615 \div 5$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



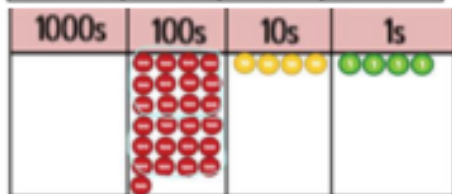
Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Long division using place value counters
 $2544 \div 12$

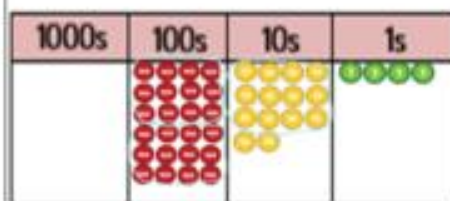


We can't group 2 thousands into groups of 12 so will exchange them.



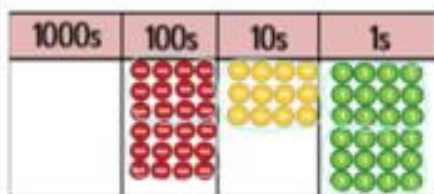
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 0.2 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

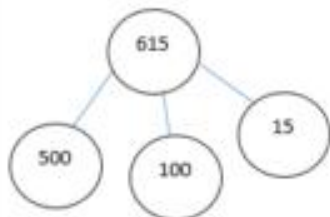


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?



